Hydrodynamics and Mass Transfer Characteristics of a Scheibel Extractor

Part II: Backmixing and Stage Efficiency

The transfer of acetone between the toluene phase (dispersed) and the aqueous phase (continuous) has been measured along the length of a four-stage Scheibel column packed with material preferentially wetted by the dispersed phase. (A Scheibel stage is defined as one mixing section plus one packing section.) The column concentration profiles were obtained by applying a new technique developed for this investigation.

A parameter model-fitting exercise has been developed to select the most appropriate mathematical model that quantifies the extent of backmixing in the dispersed and continuous phases and evaluates the stage efficiency.

Overall dispersed phase mass transfer coefficients were estimated using the drop size and holdup data presented in Part I, and these have been compared with the published correlations based on single drop mechanisms.

J. C. BONNET
and
G. V. JEFFREYS
University of Aston in Birmingham,
United Kingdom

SCOPE

It is well known that axial mixing or backmixing has a significant effect on the performance of an extraction column. Several investigators have successfully described axial mixing in the rotating disc contactor (RDC) Strand et al., 1962) and Oldshue-Rushton (Bibaud and Treybal, 1966) columns by the eddy diffusivity concept introduced by Sleitcher (1959). Their research produced very valuable correlations of the eddy diffusivities in the continuous and dispersed phases. On the other hand, studies of axial mixing in Scheibel extractors have not hitherto been studied to any great extent. Only Gelperin (1967) investigated mixing in a 0.056 m dia. 0.50 m long Scheibel column. This work was mainly devoted to the study of mixing in an RDC and in a modified Scheibel column containing no packing, and only the continuous phase axial mixing diffusivity was determined by the input-output tracer analysis techniques under nonmass transfer conditions.

Therefore the objective of the present study was the development of a suitable model that will enable experimental mass transfer data to be applied to quantify the nonideal flow parameters and evaluate realistic mass transfer coefficients in this type of contactor.

CONCLUSIONS AND SIGNIFICANCE

Axial concentration profiles presented in this paper have been interpreted in terms of the discrete stagewise model, i.e., cells in series with backmixing, and the conclusions from the analysis of the mass transfer data and the model fitting were:

1. Both phases inside the 5.0 cm height agitated compartment are completely mixed.
2. No significant amount of mass transfer takes place inside the knitted mesh pad when these are wetted by the dispersed phase. The major part of the solute transfer within a Scheibel stage occurs in the region of the impeller.
3. The cells in series with backmixing model represents the mass transfer behavior of the Scheibel column very well. A substantial deviation from plug flow was found in the dispersed phase, contradicting the belief that the flow profile in the dispersed phase is usually less important than that of the continuous phase in extractors with discrete stages.
4. Experimental overall dispersed phase mass transfer coefficients based on completely mixed stages were larger than the corresponding theortical coefficients calculated on the basis of a single drop mechanism.

INTRODUCTION

The overall effect of backmixing in liquid-liquid extractions columns is a combination of several phenomena that will vary according to the type of contactor and the flow conditions within the equipment. Backmixing or axial mixing in the continuous phase can be considered the sum of two effects. The first is a true turbulent and molecular diffusion in the axial direction characterized either by an eddy diffusivity or a backflow coefficient, which is

---

J. C. Bonnet is currently at Escuela de Ingenieria Quimica, Universidad Central de Venezuela, Caracas, Venezuela.

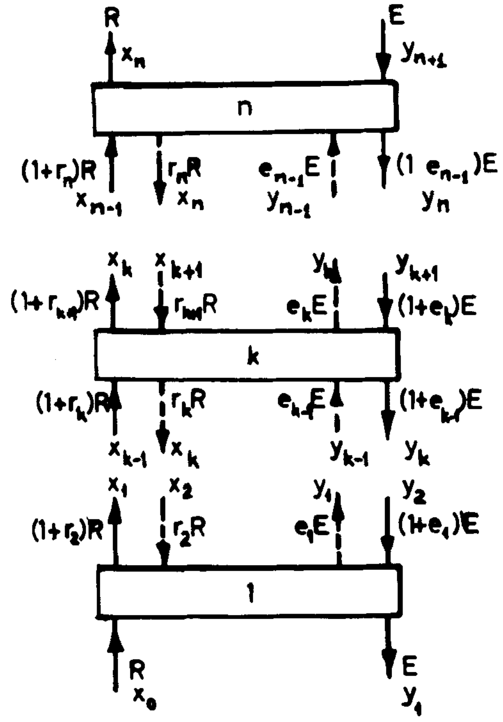

Figure 1. Model of n cells in series with backflow.

usually determined by upstream tracer observations. This effect may be manifested by vertical circulation currents, entrainment of the continuous phase by the drops, or forced backflow of part of the phase due to the turbulence in the column. The second effect is a spreading effect that is predominant in the forward direction. This may be caused by a nonuniform velocity across a section of the column, a channeling situation, or a Taylor-type diffusion. This effect is significant in columns having no or very little agitation. Although turbulent effects and drop entrainment may be important for the dispersed phase, a dominating contribution to the total axial mixing of this phase is the one caused by the redistribution of residence times of the drop swarms, resulting from the distribution of the drop sizes and therefore the distribution in the velocities of the drops.

The determination of the total axial mixing can be accomplished (1) from axial concentration profiles and (2) by stimulus-response techniques. The first method was adopted in this investigation and requires withdrawal of a number of samples of both phases along the length of the extractor during steady state mass transfer operation. The axial mixing parameters can then be evaluated by matching the experimental concentration profiles with theoretical values computed from an appropriate mathematical model.

MATHEMATICAL MODELS AND OPTIMIZATION

The eddy diffusional model has been proposed for estimating axial mixing in differential contacting equipment where radial concentration gradients can be neglected and the cell-backmixing model has been similarly applied to stagewise contactors. Figure 1 shows schematically the cells in series with backmixing model; the calculation procedure for this model has been well described by Mecklenburgh and Hartland (1975).

Application of the diffusional model requires the solution of a set of nonlinear ordinary differential equations, while the stagewise model calls for the solution of a set of nonlinear difference equations. Analytical solution can be obtained in either model by assuming constant axial mixing along the contactor, constant flow rates of the phases, linear or quadratic equilibrium relationship, and a constant volumetric overall mass transfer coefficient. If these assumptions are unacceptable, a numerical solution must be obtained, which, in the case of the diffusional model, requires the substitution of the differential terms by finite-difference approximations.

The Prochazka and Landau (1963) version of the cell backmixing model was applied in this study, and it will be seen (in Figure 1) that flow nonidealities are represented by the coefficients of backmixing, r_k and e_k, for the raffinate and extract phases, respectively. They are defined as the relative amount of entrainment of the raffinate and extract that flow in a direction opposite to that of the corresponding main phase flow. It is assumed that the composition of the entrained stream is the same as the composition of the main stream from the particular stage.

The kinetics of the mass transfer process are characterized indirectly by means of stage efficiency η; its functional relationship with the mass transfer coefficient depends on the model selected to represent the flow behavior within the stage. All the assumptions needed for an analytical solution can be relaxed and therefore axial variation of the backmixing coefficients, stage efficiency or mass transfer coefficient, and the equilibrium parameters can be easily introduced into the model. For an n stage countercurrent extractor, the model requires the simultaneous solution of a set of $2n$ difference equations arising from the application of a solute material balance and a kinetic expression to each stage. For the case of immiscible solvents a material balance on the solute for the kth stage gives

$$\begin{aligned}(1 + r_k + r'_{k+1}1)\alpha x'_{k-1} + (1 + e_k + e_{k-1})y'_{k+1} \\ = (1 + r_k + r_{k+1})\alpha x_k + (1 + e_k + e_{k-1})y_k \quad (1)\end{aligned}$$

where

$$x'_{x-1} = \frac{(1 + r_k)x_{k-1} + r_k + 1x_{k+1}}{(1 + r_k + r_k + 1)} \quad (2)$$

$$y'_{k+1} = \frac{(1 + e_k)y_{k+1} + e_{k-1}y_{k-1}}{(1 + e_k + e_{k-1})} \quad (3)$$

A stage efficiency corresponding to the kth stage, η_k, is defined as

$$\eta_k = \frac{x'_{k-1} - x_k}{x'_{k-1} - x^*_k} = \frac{y_k - y'_{k+1}}{y^*_k - y'_{k+1}} \quad (4)$$

where x^{**}_k and y^{**}_k are equilibrium concentrations

$$y^{**}_k = I_k x^*_k + J_k \quad (5)$$

that satisfy the following material balance:

$$\frac{y^*_k - y'_{k+1}}{x_{k-1} - x_k} = \frac{\alpha(1 + r_k + r_{k+1})}{(1 + e_k + e_{k-1})} \quad (6)$$

Algebraic manipulation of the above equations leads to the set of governing equations:

$$\begin{aligned}(1 + r_k)x_{k-1}\alpha + e_{k-1}y_{k-1} - (1 + r_k + r_{k+1})x_k\alpha \\ - (1 + e_k + e_{k-1})y_k + r_{k+1}x_{k+1}\alpha + (1 + e_k)y_{k+1} = 0 \quad (7)\end{aligned}$$

$$\begin{aligned}(1 - \eta_k)(1 + r_k)x_{k-1} - \frac{1 + (1 - \eta_k)F'_K}{(1 + F'_K)}(1 + r_k + r_{k+1})x_k \\ + \frac{\eta_k(1 + r_k + r_{k+1})}{I_k(1 + F'_K)}y_k + (1 - \eta_k)r_{k+1}x_{k+1} \\ = \frac{\eta_k(1 + r_k + r_{k+1})}{I_k(1 + F_k)}J_k \quad (8)\end{aligned}$$

where F'_k is a modified extraction factor defined as

$$F'_k = \frac{\alpha(1 + r_k + r_{k+1})}{I_k(1 + e_k + e_{k-1})} \quad (9)$$

When applying Eqs. 7 and 8 to the first and last stages, the following boundary conditions must be introduced:

$$r_1 = r_{n+1} = e_o = e_n = 0 \quad (10)$$

The above set of equations can be written in matrix notation

$$A \cdot X = G \quad (11)$$

where A is the concentration coefficients matrix $[(2n) \times (2n + 2)]$;

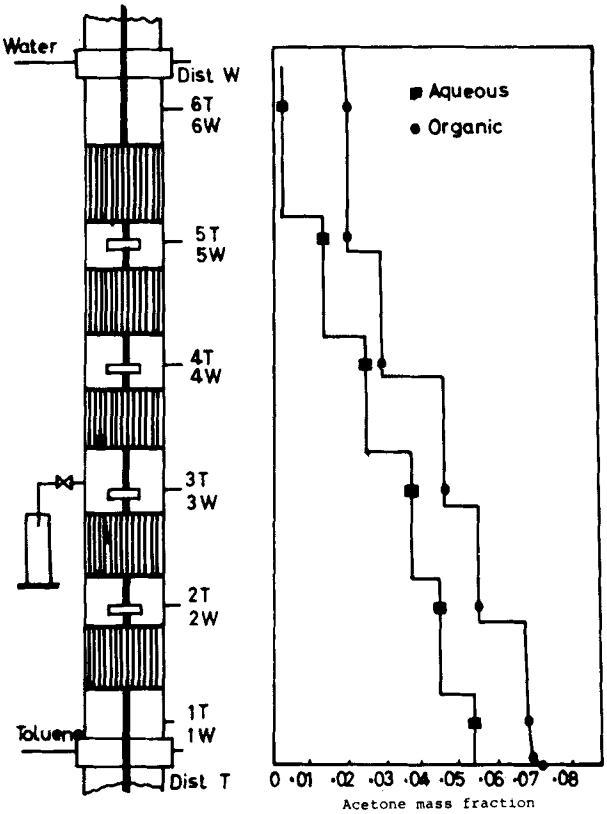

Figure 2. Concentration profiles (experiment 2).

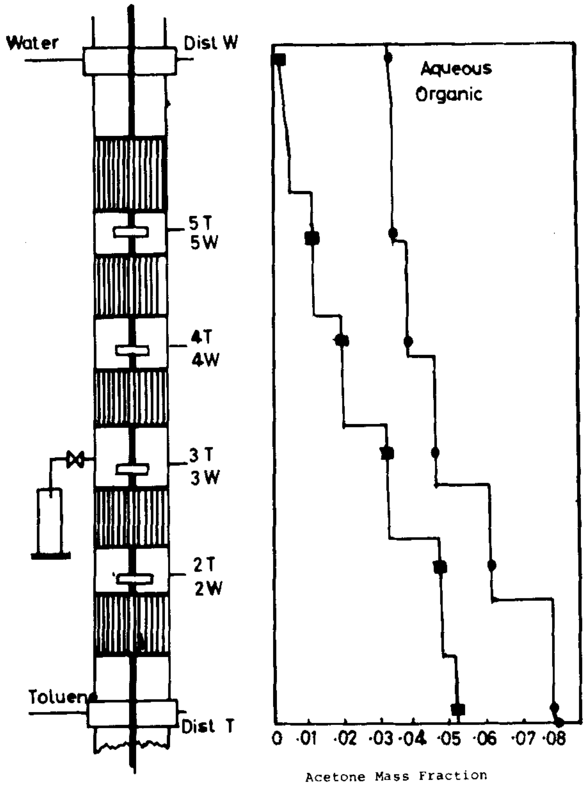

Figure 3. Concentration profiles (experiment 9).

X is the concentration vector $(x_o, x_1, y_1, \ldots . x_w, y_n, y_{n+1})$ of dimension $(2n + 2)$; and G vector $(2n)$ on the right-hand side of the kinetic relation equation, 8. The elements of the matrix A and vector G are functions of η_k e_k, r_k, I_k, J_k, and a and have been derived by Bonnet (1982).

Optimization

A regression analysis was implemented to evaluate the nonideal flow parameters of the quantities that express the rate of mass transfer in the above models and the best values of these model parameters are those that minimize an objective function of the least-squares type:

$$f = \sum_{i=1}^{m} (P_i - p_i)^2 = \sum_{i=1}^{m} R_i(b)^2 \tag{12}$$

where P_i is the experimental value of the dependent variable for the ith observation, p_i the value of the P_i predicted by $f(z,b)$, for the ith observation and the b vector of the parameters. The term $\Sigma R(b)^2$ is the residual for the ith observation. The stagewise model is nonlinear with respect to these parameters, requiring for its solution an iterative method. The Marquardt algorithm (Marquardt, 1963) has been successfully applied in this study.

The objective function for the curve fitting was defined in terms of Eq. 11 as follows:

$$\phi = \|X_1 - X_1\|^2 \tag{13}$$

where $X_1 = A_1^{-} \cdot G_1$ is the predicted concentration vector $(x, y, \ldots . x_n, y_n)^T$, with x_o and y_{n+1} being the feed stage concentrations included in the vector G_1 in order to obtain a square coefficient matrix A_1 of dimension $(2n \times 2n)$. The solution vector X_1 of the set of linear algebraic equations was obtained using the Crout factorization method, and Eq. 13 was minimized with respect to the vector of the parameters $\beta^T = (\eta_k, e_k, r_k)^T$.

A theoretical five-cell concentration profile was used to study the influence of a number of the parameters in the model and the data error of the solution was optimized. The conclusions are briefly summarized as follows:

1. For the model with the largest number of parameters $(2n)$, different solutions were obtained for different starting points. Since the theoretical and calculated profiles agree exactly up to the sixth significant digit, it was not possible to find a criterion of choice of the best solution vector. The greatest disagreement between different solution vectors is in the values of the backmixing parameters; the efficiencies are less affected.
2. Only the models with five (axial variation in efficiency only) and three (in axial variation) parameters gave the same solution vector for different starting points.
3. The estimations of the backmixing coefficients are affected more by the number of parameters in the model and the data errors than the efficiency. Concentration profiles with random data errors in the range 0–5% lead to unacceptable errors in the estimated values of the parameters.

More details about the properties of the models and their solutions are given by Bonnet (1982) and Slavickova et al. (1978).

EXPERIMENT

A detailed description of the apparatus employed for this study has been presented in Part I. Figures 2 and 3 show schematically columns 1 and 2 with the sampling points and holdup values clearly indicated. Typical experimental concentration profiles are shown alongside the diagrams of the columns and were drawn to represent the real axial concentration variation of each phase along the columns. The concentration profiles were measured using a single phase sampling technique developed in this investigation by Bonnet and Jeffreys (1982).

The acetone content in the samples of the continuous phase was determined by refractive index measurements and that in the dispersed phase by gas-liquid chromatography using the interval standard method (Littlewood, 1962) with benzene as internal standard. Acetone concentrations in either organic or aqueous feed streams were always less than 0.12 mass acetone per unit mass of solvent; therefore total immiscibility of the phases can be assumed. The equilibrium data of Othmer et al. (1941) were used.

The experimental program was devised to interpret the experimental concentration profiles with those described in the mathematical model. Four Scheibel stages were considered appropriate for this purpose, and the mass transfer experiments performed followed the 2^4 factorial design presented in Table 2 of Part I.

TABLE 1. RESULTS OF FACTORIAL EXPERIMENT

Run	Mass Transfer Direction	Pad Height	V_D	N	% MBC	θ_D	d_{32} (st. dev)	a	η	r	e	K_D
1	D → C	0.06	1.28×10^{-3}	6.66	2.50	0.045	2.53×10^{-3} (1.30×10^{-3})	1.06×10^{-2}	0.393	0.485	0.75	5.58×10^{-5}
2			1.28×10^{-3}	10.00	2.10	0.055	2.22×10^{-3} (1.05×10^{-3})	1.48×10^{-2}	0.606	0.289	0.641	11.40×10^{-5}
3			1.94×10^{-3}	6.66	2.90	0.068	2.28×10^{-3} (1.5×10^{-3})	1.44×10^{-2}	0.412	0.969	0.605	9.06×10^{-5}
4			1.94×10^{-3}	10.00	4.70	0.074	2.76×10^{-3} (1.43×0^{-3})	1.61×10^{-2}	0.626	0.017	0.946	13.73×10^{-5}
5	C → D		1.28×10^{-3}	6.66	0.76	0.080	1.43×10^{-3} (0.54×10^{-3})	3.34×10^{-2}	0.655	0.823	0.290	5.80×10^{-5}
6			1.28×10^{-3}	10.00	3.10	0.142	0.82×10^{-3} (0.26×10^{-3})	10.32×10^{-2}	0.801	0.775	0.091	3.33×10^{-5}
7			1.94×10^{-3}	6.66	7.00	0.100	2.04×10^{-3} (0.89×10^{-3})	2.93×10^{-2}	0.633	0.621	0.613	8.33×10^{-5}
8			1.94×10^{-3}	10.00	11.00	0.171	1.27×10^{-3} (0.47×10^{-3})	8.03×10^{-2}	1.085	0.798	−0.010	—
9	D → C	0.03	1.28×10^{-3}	6.66	3.00	0.044	2.96×10^{-3} (1.55×10^{-3})	0.89×10^{-2}	0.379	1.569	0.050	11.00×10^{-5}
10		0.03	1.28×10^{-3}	10.00	1.71	0.057	2.29×10^{-3} (1.10×10^{-3})	1.49×10^{-2}	0.682	0.439	0.441	17.83×10^{-5}
11			1.94×10^{-3}	6.66	2.40	0.072	3.86×10^{-3} (2.25×10^{-3})	1.12×10^{-2}	0.451	0.730	0.199	13.83×10^{-5}
12			1.94×10^{-3}	10.00	2.51	0.080	3.28×10^{-3} (1.76×10^{-3})	1.46×10^{-2}	0.593	0.446	0.125	15.50×10^{-5}
13	C → D		1.28×10^{-3}	6.66	1.52	0.060	1.78×10^{-3} (0.77×0^{-3})	2.02×10^{-2}	0.574	0.827	0.143	8.16×10^{-5}
14			1.28×10^{-3}	10.00	4.82	0.124	1.32×10^{-3} (0.62×10^{-3})	5.63×10^{-2}	0.713	0.922	−0.01	4.73×10^{-5}
15			1.94×10^{-3}	6.66	4.30	0.116	2.24×10^{-3} (1.18×10^{-3})	3.10×10^{-2}	0.494	1.251	0.229	6.70×10^{-5}
16			1.94×10^{-3}	10.00	2.10	0.158	1.06×10^{-3} (0.43×10^{-3})	8.94×10^{-3}	0.896	0.455	0.358	15.41×10^{-5}
10-1			1.28×10^{-3}	10.00	4.00	0.062	—	—	0.785	0.465	0.557	
14-1			1.28×10^{-3}	10.00	1.10	0.116	—	—	0.934	0.906	−0.029	

In all the runs $V_c = 1.02 \times 10^{-3}$ m/s.

RESULTS AND DISCUSSION

In all the experiments and data collection for the factorial design, the impellers were positioned at approximately $D_I/5$ cm from the upper surface of the pads. The position of the interface at the top of the column was maintained at about 20 cm from the top distributor plates and carefully monitored at all times in order to avoid random variations that would magnify any experimental error.

Sixteen concentrations profiles were obtained, two of which are illustrated in Figures 2 and 3. They were evaluated in terms of the five-stage backmixing model schematically presented in Figure 1. The three-parameter model was fitted to the profiles using the Marquardt algorithm to minimize the sum of squares of the residuals of the objective function. Table 1 summarizes the results of the factorial experiment. The column in the table headed "MBC" gives the material balance closure percentage error for acetone. Figures 4a and 4b and 5a and 5b give an indication of the goodness of fit, which is seen to describe the behavior of the column quite well.

The iterations very slowly approached the minimum in experiments 8 and 14 and in both cases convergence was not achieved. This was most probably due to the fact that the optimum value of one of the parameters (e) was approached from a region outside the boundaries of the constraints given by

$$-0.01(\eta_k, r_k, e_k)^T < 10$$

most likely because e_{opt} lies very close to zero. Theoretical analysis of the stagewise profiles had shown that experimental and/or mathematical model errors, together with the proximity of the solution vector to the origin, can lead to a negative value of the backmixing parameters.

The optimum value of the stage efficiency for experiment 8 shown in Table 1 was found to be slightly greater than 1, but to assume that a significant amount of mass transfer took place inside the pads would not be justifiable, as it can be seen, from the result of the extract phase backmixing parameter (e,0), that a significant

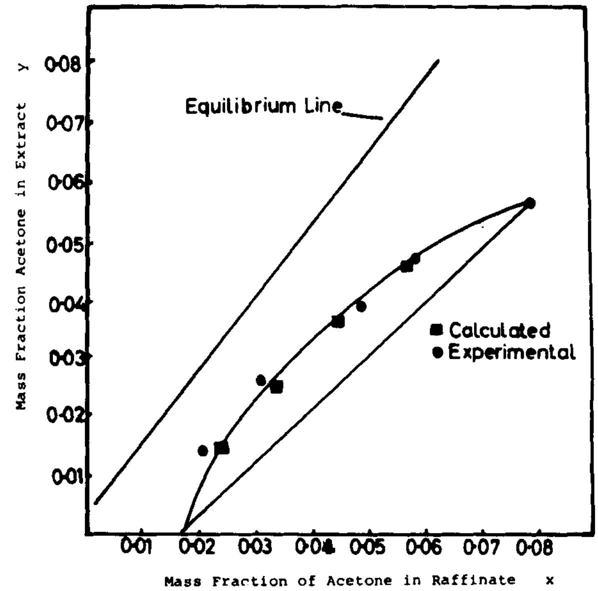

Figure 4a. Comparison of experimental and calculated results (experiment 2).

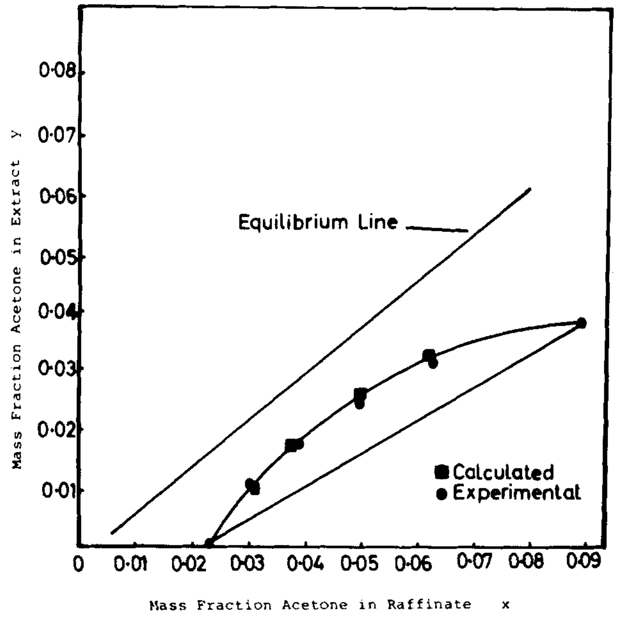

Figure 4b. Comparison of experimental and calculated results (experiment 7).

error affects the profile and could be responsible for the large value of this parameter. Furthermore, the mass balance closure (MBC) for this run was 11% and could influence the result.

Statistical Analysis: Backmixing Parameters

The data obtained for the raffinate parameter, for experiments with mass transfer direction dispersed to continuous phase and the extract backmixing parameter, e, for the experiments with the opposite direction of mass transfer, were analyzed in terms of an analysis of variance using the Yates method to calculate main effects and interactions. The error variance was estimated from the three- and four-factor interaction, and the results are given in Table 2. The analysis indicated that only the agitator speed and the mass transfer direction have a significant influence on the value of the dispersed phase nonideal flow parameter. The magnitude and sign of these effects were calculated to be

$$\text{Agitator speed } (A) = 0.4280$$
$$\text{Mass transfer direction} = 0.4048$$

and since interactions are not significant, both main effects are independent. The decrease in the value of the backmixing parameter with increasing agitator speed is the result of a more uniform drop size distribution, as indicated by the decrease in the standard deviations of the distribution listed in Table 1. The same result can be achieved by changing the direction of mass transfer from dispersed to continuous to continuous to dispersed phase (e.g., runs 2 and 6, runs 10 and 14, etc.). With the exception of run 16, the combination of high agitator speed and mass transfer direction continuous to dispersed phase leads, practically, to an ideal flow behavior of the dispersed phase (backmixing parameter $\simeq 0$).

The result of a nonsignificant influence of the dispersed phase superficial velocity (factor D) was surprising. It was expected that an increase in V_D may increase the nonideality of the dispersed

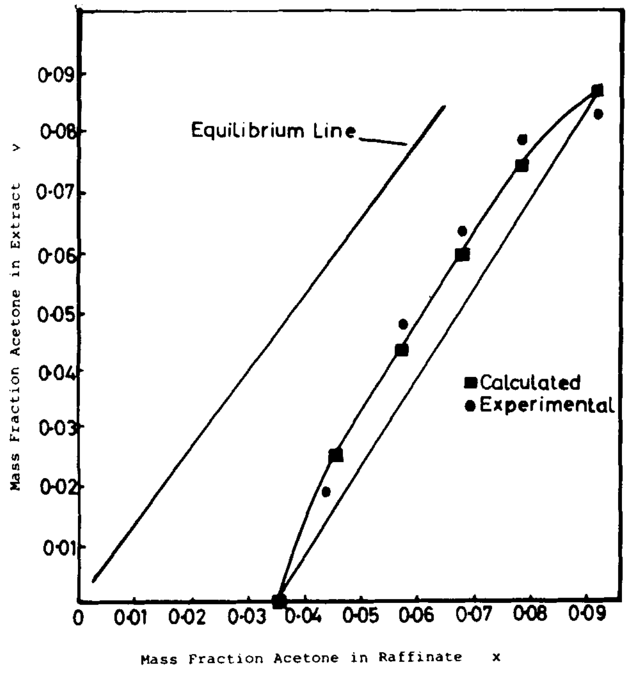

Figure 5a. Comparison of experimental and calculated results (experiment 12).

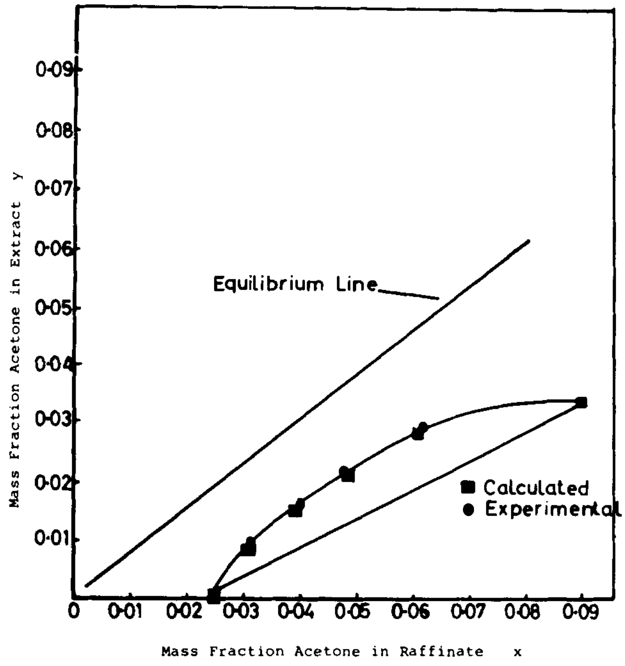

Figure 5b. Comparison of experimental and calculated results (experiment 15).

TABLE 2. ANALYSIS OF VARIANCE: DISPERSED PHASE BACKFLOW PARAMETER

Source of Variation	Deg of Freedom	Mean Square	F
A	1	0.7357	7.12*
B	1	0.0898	0.86
C	1	0.6556	6.34**
D	1	0.0000	0.00
AB	1	0.0147	0.142
AC	1	0.1951	1.88
AD	1	0.0000	0.00
BC	1	0.1881	1.82
BD	1	0.0395	0.382
CD	1	0.0965	0.934
ABC	1	0.0743	
ABD	1	0.3033	
ACD	1	0.0024	
BCD	1	0.0910	
ABCD	1	0.0456	
Error	= 5	$= \frac{0.5166}{5} = 0.1033$	

F(1,5) = 6.61 at 5% level.
F(1,5) = 4.06 at 10% level.
* Significant.
** Probable significance.

TABLE 3. ANALYSIS OF VARIANCE: CONTINUOUS PHASE BACKFLOW PARAMETER

Source of Variation	Deg. of Freedom	Mean Square	F
A	1	0.0251	0.51
B	1	0.0617	1.26
C	1	0.7131	14.60*
D	1	0.0116	0.23
AB	1	0.1253	2.56
AC	1	0.2040	4.17**
AD	1	0.1159	2.37
BC	1	0.2227	4.56**
BD	1	0.0460	0.94
CD	1	0.0490	1.00
ABC ABD ACD BCD } Error ABCD	1 1 1 1 } = 5 1	0.0035 0.1140 0.0001 0.0789 } $= \frac{0.2442}{5} = 0.0488$ 0.0477	

F(1,5) = 16.3 at 1% level.
F(1,5) = 6.61 at 5% level.
F(1,5) = 4.06 at 10% level.
* Significant.
** Probable Significance.

TABLE 4. ANALYSIS OF VARIANCE: STAGE EFFICIENCY

Source of Variation	Deg. of Freedom	Mean Square	F
A	1	0.2527	33.73*
B	1	0.0115	1.53
C	1	0.1833	24.43*
D	1	0.0092	1.22
AB	1	0.0000	0.00
AC	1	0.0044	0.00
AD	1	0.0100	1.33
BC	1	0.0202	2.69
BD	1	0.0028	0.37
CD	1	0.0074	0.98
ABC ABD ACD BCD } Error ABCD	1 1 1 1 } = 5 1	0.0003 0.0026 0.0330 0.0007 } $= \frac{0.0374}{5} = 0.0075$ 0.0008	

F(1,5) = 6.61 at 5% level.
F(1,5) = 16.30 at 1% level.
* Highly significant.

phase flow by affecting the spread of the drop size distribution and the appearance of phase bypass. Since the dispersed phase backmixing data presents the largest error variance, it can be concluded that the change in the level of V_D was not large enough, and the probable existence of a significant effect of this factor is marked by the large total error (true experimental plus lack of fit).

Table 3 presents the results of the analysis of variance of the data corresponding to the continuous phase backmixing parameter. The analysis shows that the interactions (agitator speed × mass transfer direction) and (pad height × mass transfer direction) are significant at the 10% level.

If very little or no reliance is placed on the 10% level of significance, the only significant mean square is the one associated with the factor (mass transfer direction). The positive sign of this main effect indicates that an increase in the backmixing parameter results from a change in the mass transfer direction from dispersed to continuous to continuous to dispersed phase. Since tracer tests carried out in columns 1 and 2 showed that physical backflow or entrainment in either phase did not occur, it must be accepted that the chaneling effect of the continuous phase, created by the high continuous phase linear velocity in the packed section persisting in the vicinity of the wall of the mixing chamber, is responsible for any change in the continuous phase nonideal flow parameter. This phenomenon, which is associated with a poor radial diffusivity, depends entirely on the intensity of the turbulence in the continuous phase and its distribution in the vicinity of the chamber wall. A high level of turbulence increases the radial diffusivity and eliminates the channeling at the wall. It has been shown (Lats and Frishman, 1974) that the intensity turbulence decreases as drop concentration or holdup increases. The data obtained in the factorial experiment indicated that, for equal agitator speeds, the holdup increases in the range of 150 to 200% when solute transfer direction changes from dispersed to continuous to continuous to dispersed phase; therefore, a positive change in the continuous phase backmixing parameter would be expected.

Using the above relationship between intensity of turbulence and holdup; it is reasonable to expect a significant influence of impeller speed on the backmixing parameter. Such a situation did not occur, and the variation of the backmixing parameter with agitator speed, N, is seen to be completely random, leading to a nonsignificant effect. A tentative explanation could be that increasing N increases the intensity of turbulence and holdup; since both have opposite effects on the continuous phase backmixing, they may cancel out each other.

Statistical Analysis: Stage Efficiency

The stage efficiencies given in Tabe 1 were studied in an analysis of variance. Table 4 shows that only agitator speed and the mass transfer direction were found to be significant. An important finding is that packing pad height, within the levels studied, does not influence η, and therefore it can be concluded that no appreciable mass transfer took place inside the packing. The magnitude of the significance effects are

$$\text{Agitator speed } (A) = 0.2513$$
$$\text{Mass transfer direction } (C) = 0.2141$$

The combined effect of increasing the interfacial area as agitator speed increases and decreasing the dispersed phase backmixing parameters as mass transfer direction changes from dispersed to continuous to continuous to dispersed explains the positive effect of the above factors over the stage efficiency.

Overall Mass Transfer Coefficient

The overall mass transfer coefficient K_D, based on the dispersed phase, are listed in Table 5 and were calculated using the equation

$$\eta^k = \frac{(1 + F'_k)}{(1 + F'_k) + 1} \cdot \frac{K_D(aV_K)}{R1 + r_k + r_{k+1})} \quad (14)$$

This is based on perfect mixing of the phases within a stage. Due to the complexities of the drop formation process as shown in Figure 2, (b-2) of Part I, it is extremely difficult to measure or predict the total interfacial area for mass transfer in the impeller region. Therefore it was assumed that the specific interfacial area a was equal to the surface area of all free drops and calculated using the Sauter mean drop diameter d_{32} and holdup data listed in Table 1.

For comparison, theoretical overall mass transfer coefficients were calculated using well-known correlations derived from single drop mass transfer studies. Individual dispersed phase mass transfer coefficients were calculated using the correlation proposed by Newmann (1931) and later simplified by Vermuelen (1953) for stagnant drops; the Kronig and Brink correlation (1960) for circulating drops and the Rose and Kintner (1966) correlation for oscillating drops. The continuous phase coefficients were calculated using the correlation given by Garner et al. (1960) for the case of stagnant and circulating drops. Both of these coefficients were substituted in the usual equation of additivity of phase resistance to give the overall coefficient.

Table 5 lists the experimental and theoretical overall mass

TABLE 5. EXPERIMENTAL AND THEORETICAL OVERALL DISPERSED PHASE MASS TRANSFER COEFFICIENT

Run	d_{32}	K_D Exp., m/s	Stagnant Drop	K_D (theoret.) Circ. Drop, m/s	Oscillating Drops	
					$n = 2$	$n = 3$
1	2.53×10^{-3}	5.58×10^{-5}	1.61	3.75×10^{-5}	6.61	6.98
4	2.76×10^{-3}	13.73×10^{-5}	1.51	3.52×10^{-5}	6.18	6.46
5	1.43×10^{-3}	5.80×10^{-5}	1.51	3.51×10^{-5}	7.14	7.45
6	0.82×10^{-3}	3.33×10^{-5}	—	3.64×10^{-5}	7.63	7.86
7	2.04×10^{-3}	8.33×10^{-5}	1.45	3.37×10^{-5}	6.39	6.70

transfer coefficients. In the oscillating single drop case, theoretical coefficients for two different modes of oscillation were calculated.

If drop size is the criterion used to select the appropriate single drop mechanism, the results of Table 5 show that all the experimental coefficients are larger than the corresponding theoretical. Drops in run 6 were expected to behave as rigid spheres, in run 4 as circulating-oscillating drops, and in run 7 as circulating drops. The result is that the predictions tend to underestimate the experimental values, which can be explained in terms of the violent nature observed during drop formation. The most likely stagnant drops of run 6 probably began their free rise existence with some degree of oscillation induced by the breakup process, so that mass transfer coefficients estimates based on completely rigid drops would be on the low side. Similarly, the circulating-oscillating drops of run 4 probably experienced enhanced oscillation due to the violence of the swirling jet breakup compared with a free oscillating drop created at a nozzle.

ACKNOWLEDGMENT

One of the authors, J. C. Bonnet would like to thank the Consejo de Desarrollo Cientifico y Humanistico and Projecto Unesco VEN-31, both of the Universidad Central de Venezuela, and FONNINVES, Venezuela, for the financial support during this work.

NOTATION

A = constants in Eqs. 14 and 15
A = specific interfacial area, m^2/m^3
a = specific interfacial area, m^2/m^3
C^* = equilibrium concentration, kg/m^3
C_x = reaffinate phase solute concentration, kg/m^3
C_y = extract phase solute concentration, kg/m^3
d_{32} = Sauter mean drop diameter, m
e_k = fractional backflow coefficient in the kth stage, exact phase
E = solute-free extract phase mass flow rate, kg/s
e_x = axial dispersion coefficient phase, $f(E_x,\theta_x)$, m^2/s
e_y = axial dispersion coefficient of extract phase, $f(E_y,\theta_y)$, m^2/s
F = extraction factor = $V_x \cdot m/V_y$
F_k = modified extraction factor defined by Eq. 9
I_k = apparent slope of equilibrium line, defined by Eqs. 4, 5, and 6, mass ratio units
J_k = apparent intercept of equilibrium line, defined by Eqs. 4, 5, and 6, mass ratio units
k_x = raffinate phase overall mass transfer coefficient, m/s
K_D = dispersed phase overall mass transfer coefficient, m/s
L = height of active extraction part of a column
m = distribution coefficient
q = intercept of equilibrium equation, kg/m^3
R = solute-free raffinate phase mass flow rate, kg/s
r_k = fractional backflow coefficient in kth stage, raffinate phase
V_k = volume of kth Scheibel stage, m^3
V_x = raffinate phase superficial velocity based on extractor cross section, m/s
V_y = extract phase superficial velocity based on extractor cross section, m/s
x = raffinate phase solute concentration, mass acetone/mass toluene
X = raffinate phase dimensionless concentration, $C_x - (mC_{yi} + q)/C_{xi} - (mC_{yi} + q)$
y = extraction phase solute concentration, mass acetone/mass water
Y = extract phase, dimensionless concentration, $m(C_y - C_{yi})/C_{xi} - (mC_{yi} + q)$
z = height in column, m
Z = dimensionless axial column length = x/L

Greek Letters

θ_D = dispersed phase volumetric fractional holdup
θ_x = raffinate phase volumetric fractional holdup
θ_y = $(1 - \theta_x)$
α = R/E
η = stage efficiency

LITERATURE CITED

Bibaud, R. E., and R. E. Treybal, "Axial Mixing and Extraction in a Mechanically Agitated Liquid Extraction Tower," *AIChE J.*, **12**, 472 (1966).

Bonnet, J. C., "Hydrodynamics and Mass Transfer Studies in a Scheibel Extractor," Ph.D. Thesis, Aston Univ., Birmingham, England (1982).

Bonnet, J. C., and G. V. Jeffreys, "Measurement of Concentration Profiles in a Liquid-Liquid Extraction Column," *J. Chem. Technol. Biotechnol*, **33A**, No. 4, 176 (1983).

Calderbank, P. H., and J. J. Korchinski, "Circulation in Liquid Drops," *Chem. Eng. Sci.*, **6**, 65 (1956).

Garner, F. H., A. Foord, and M. M. Tayeban, *An. R. Soc. Esp. Fis. Quim.*, **546B**, 479 (1960).

Gelperin, N. L., "Axial Mixing in Agitated Extraction Columns" (trans. from Russian), *Test. Osn. Kihm. Teknol.*, **1**, 666 (1967).

Kronig, R., and J. C. Brink, "On the Theory of Extraction from Falling Droplets," *Appl. Sci. Res.*, **A2**, 142 (1960).

Lats, M. K., and F. A. Frisham, "Fluid Dynamics" (trans. from Russian), **8**, 304 (1974).

Littlewood, A. B., *Gas Chromatography*, 2nd Ed., Press, London (1962).

Marquardt, D., "An Algorithm for Least Squares Estimation of Non-Linear Parameters," *J. Soc. Ind. Appl. Math.*, **11**, 431 (1963).

Mecklenburgh, J. C., and S. Hartland, *The Theory of Backmixing*, Wiley, New York (1975).

Newmann, A. B., "The Drying of Porous Solid: Diffusion and Surface Emission Equations," *Trans. Am. Inst. Chem. Engrs.*, **27**, 203 (1931).

Othmer, D. F., R. E. White, and E. Truegar, "Liquid-Liquid Extraction Data," *Ind. Eng. Chem.*, **32**, 1240 (1941).

Pratt, H. R. C., "A Simplified Analytical Design Method for Differential Extractors with Backmixing," *Ind. Eng. Chem. Proc. Des. Devel.*, **14**, 74 (1975).

Prochazka, J., and J. Landau, "Backmixing and Efficiency of Continuous Stagewise Countercurrent Extraction Process," *Coll. Czech. Chem. Comm.*, **28**, 1927 (1963).

Rose, P. M., and R. C. Kintner, "Mass Transfer from Large Oscillating Drops," *AIChE J.*, **12**, 530 (1966).

Slavickova, A., G. Augelor, A. Hyberger, and J. Prochazka, "Countercurrent Extraction with Backmixing—Evaluation of Parameters," *Chem. Czech. Chem. Comm.*, **43**, 2682 (1978).

Sleicher, C. A., "Axial Mixing and Extraction Efficiency," *AIChE J.*, **5**, 145 (1959).

Strand, C. P., R. B. Olney, and G. H. Ackerman, "Fundamental Aspect of RDC Performance," *AIChE J.*, **8**, 252 (1962).

Vermeulen, T., "Theory for Irreversible and Constant Pattern Solid Diffusion," *Ind. Eng. Chem.*, **45**, 1664 (1953).

Manuscript received Jan. 4, 1984, and accepted July 2.